Propogation, Reflection and Refraction of the Electromagnetic Waves :-

The interaction between electromagnetic wave and matter is the interaction between two groups of physical quantities i.e electric field $\vec{E}(\vec{r},t)$ and magnetic field $B(\vec{r},t)$ also electric susceptibility χ_e and magnetic susceptibility χ_m and electrical conductivity σ.

Refection and refraction of electromagnetic waves are an interesting phenomenon and have several applications such as developing radar systems, developing photonic crystals and investigating biological materials.

When the medium of propogation of em wave is vacuum then, the value of medium permitivity is equal to vacuum permitivity ϵ_0 and also vacuum permeability μ_0.
where $\epsilon_0 = 8.85 \times 10^{-12}\ C^2/N.m^2$

and $\mu_0 = 4\pi \times 10^{-7}\ T.m/A.$

When em wave propogates in non-vacuum medium then the value of ϵ is

$$\epsilon = \epsilon_0 \epsilon_r$$

Where ϵ_r is the relative permitivity of the medium.

Using vector identity

$$(\vec{\nabla}\times\vec{B})\times\vec{B} = (\vec{B}\cdot\vec{\nabla})\vec{B} - \vec{\nabla}(\vec{B}\cdot\vec{B})$$

linearizing $\vec{B}$ on R.H.S.

$$(\vec{\nabla}\times B)\times B = (B_0\cdot\nabla)B_1 - \nabla(B_0\cdot B_1)$$

Putting the value in above eqs of motion and also linearized the equation

$$\frac{dV_{d1}}{dt} = -\frac{\nabla}{\rho_{0d}}\left[(P_{0e}+P_{0i})\frac{5}{3} + \frac{\alpha_1}{3}(T_{0e}^4 + T_{0i}^4)\frac{8}{3}\right]\left(\frac{n_{1d}}{n_{0d}}\right)$$

$$+\frac{1}{4\pi\rho_{0d}}\left[(B_0\cdot\nabla)B_1 - \nabla(B_0\cdot B_1)\right] - \nabla\psi_1$$

$$\frac{dV_{d1}}{dt} = \frac{-\nabla}{\rho_{0d}}\left[(P_{0e}+P_{0i})\frac{5}{3} + \frac{\alpha_1}{3}(T_{0e}^4 + T_{0i}^4)\frac{8}{3}\right]\left(\frac{n_{1d}}{n_{0d}}\right)$$

$$+\frac{1}{4\pi\rho_{0d}}(B_0\cdot\nabla)B_1 - \frac{1}{4\pi\rho_{0d}}\nabla(B_0\cdot B_1) - \nabla\psi_1$$

where V_{sd}^2 is dust acoustic velocity

$$V_{sd}^2 = \frac{5}{3}\left(\frac{P_{0e}+P_{0i}}{\rho_{0d}}\right)$$

∵ basic equation $V = \sqrt{\frac{\gamma P}{\rho}}$

And V_{rd}^2 is dust radiation velocity

$$V_{rd}^2 = \frac{8\alpha_1}{9}\left(\frac{T_{0e}^4 + T_{0i}^4}{\rho_{0d}}\right)$$

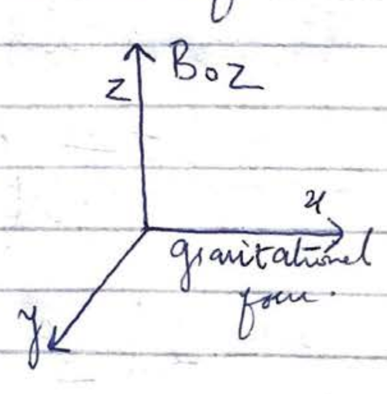
where V_d^2 is the Jeans's velocity

$$V_d^2 = V_{sd}^2 + V_{rd}^2$$

So equation of motion becomes

$$\boxed{\frac{dV_{d1}}{dt} = -\nabla V_d^2\left(\frac{n_{1d}}{n_{0d}}\right) + \frac{1}{4\pi\rho_{0d}}(B_0\cdot\nabla)B_1 + \frac{1}{4\pi\rho_{0d}}\nabla(B_0\cdot B_1) - \nabla\psi_1}$$

we have investigated the propogation of waves by taking the magnetic field into account along with the thermal radiation and gravitational field (force). We suppose that the external magnetic field is directed along z-axis while the gravitational force along x-axis.



So the equation of motion will be

$$\frac{dV_{d1}}{dt} = \frac{1}{4\pi\rho_{0d}}\left(B_{0z}\frac{\partial}{\partial z}\right)\left(B_1\hat{i} + B_1\hat{k}\right) - \frac{1}{4\pi\rho_{0d}}\nabla\left(B_{0z}B_1\right)$$

$$-\nabla V_d^2\left(\frac{n_{1d}}{n_{0d}}\right) - \nabla\psi_1 \quad \text{——} \quad (7)$$

Taking Continuity equation and linearizing it-

$$\frac{\partial n_{d1}}{\partial t} + n_{od} \nabla \cdot V_{d1} = 0$$

Differentiating w.r.t 't'.

$$\frac{\partial^2 n_{d1}}{\partial t^2} + n_{od} \nabla \cdot \frac{\partial V_{d1}}{\partial t} = 0$$

or $$\frac{\partial^2}{\partial t^2}\left(\frac{n_{1d}}{n_{od}}\right) + \nabla \cdot \left(\frac{\partial V_{d1}}{\partial t}\right) = 0$$

Putting the value of $\frac{dV_{d1}}{dt}$ from eqs (7) in above eqs

$$0 = \frac{\partial^2}{\partial t^2}\left(\frac{n_{1d}}{n_{od}}\right) + \nabla \cdot \frac{1}{4\pi \rho_{od}}\left(B_{oz}\frac{\partial}{\partial z}\right)\left(B_1\hat{i} + B_1\hat{k}\right)$$

$$- \nabla \cdot \frac{1}{4\pi \rho_{od}} \nabla \left(B_{oz} B_{1z}\right) - \nabla \cdot \nabla V_d^2 \left(\frac{n_{1d}}{n_{od}}\right)$$

$$- \nabla \cdot \nabla \psi_1$$

or

$$0 = \frac{\partial^2}{\partial t^2}\left(\frac{n_{1d}}{n_{od}}\right) + \frac{1}{4\pi \rho_{od}} B_{oz}\left(\frac{\partial}{\partial x}\frac{\partial}{\partial z} B_{1x} + \frac{\partial^2}{\partial z^2} B_{1z}\right)$$

$$- \frac{1}{4\pi \rho_{od}} \nabla^2 \left(B_{oz} B_{1z}\right) - \nabla^2 V_d^2 \left(\frac{n_{1d}}{n_{od}}\right) - \nabla^2 \psi_1$$

As we know from geometry $\vec{\nabla} = ik$

and $\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial z}$ and $\frac{\partial}{\partial y} = 0$

So

$$\frac{1}{4\pi \rho_{od}} \nabla B_{oz} \left(\frac{\partial}{\partial z} B_{1x} + \frac{\partial}{\partial z} B_{1z} \right)$$

$$= \frac{1}{4\pi \rho_o d} B_{oz} \left(\frac{\partial}{\partial x} \frac{\partial}{\partial z} B_{1x} + \frac{\partial^2}{\partial z^2} B_{1z} \right)$$

~~then~~ Also

$$\frac{1}{4\pi \rho_{od}} \nabla^2 \left(B_{oz} B_{1z} \right)$$

$$= \frac{B_{oz}}{4\pi \rho_{od}} \left(\frac{\partial^2}{\partial x^2} B_{1z} + \frac{\partial^2}{\partial z^2} B_{1z} \right)$$

So eqs of motion will be

$$0 = \frac{\partial^2}{\partial t^2} \left(\frac{n_{1d}}{n_{od}} \right) + \frac{1}{4\pi \rho_{od}} B_{oz} \left(\frac{\partial}{\partial x} \frac{\partial}{\partial z} B_{1x} + \frac{\partial^2}{\partial x^2} B_{1z} \right)$$

$$- \frac{B_{oz}}{4\pi \rho_{od}} \left(\frac{\partial^2}{\partial x^2} B_{1z} + \frac{\partial^2}{\partial z^2} B_{1z} \right) - \nabla^2 V_d^2$$

$$- \nabla^2 \psi_1$$

or

$$\boxed{0 = \frac{\partial^2}{\partial t^2} \frac{n_{1d}}{n_{od}} + \frac{B_{oz}}{4\pi \rho_{od}} \left(\frac{\partial}{\partial x} \frac{\partial}{\partial z} B_{1x} + \frac{\partial^2}{\partial z^2} B_{1z} \right) - \nabla^2 V_d^2 - \nabla^2 \psi_1}$$

By Maxwell's equation the monopole does not exist so

$$\nabla \cdot B = 0$$

where in this case $\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial z}$

Also $\frac{\partial^2}{\partial x^2} = \overline{K_x}^2$

where $k^2 = K_x^2 + K_z^2$ $\quad (k_y = 0)$

$\because$ Wave is propogating in (x, z) region.

$$\vec{B} = \left(B_{1x}\,\hat{i} + B_{1z}\,\hat{k} \right)$$

$$\nabla \cdot B = 0$$

$$\frac{\partial}{\partial x} B_{1x} + \frac{\partial}{\partial z} B_{1z} = 0$$

or $$\frac{\partial}{\partial x} B_{1x} = - \frac{\partial}{\partial z} B_{1z}$$

Putting the values in above eqs of motion

$$0 = \frac{\partial^2}{\partial t^2} \frac{n_{1d}}{n_{0d}} - \frac{B_{0z}}{4\pi \rho_{0d}} \left(\frac{\partial^2}{\partial z^2} B_{1z} + \frac{\partial^2}{\partial \cancel{x}^2} B_{1z} \right)$$

$$- \nabla^2 V_d^2 \left(\frac{n_{1d}}{n_{0d}} \right) - \nabla^2 \psi_1$$

or $$\frac{\partial^2}{\partial t^2} \frac{n_{1d}}{n_{0d}} - \frac{B_{0z}}{4\pi \rho_{0d}} \left(-k^2 B_{1z} \right) - \nabla^2 V_d^2 \left(\frac{n_{1d}}{n_{0d}} \right) - \nabla \psi_1 = 0$$

or

$$\boxed{\frac{\partial^2}{\partial t^2}\frac{n_{1d}}{n_{0d}} + \frac{B_{0z}}{4\pi\rho_{0d}}\left(\bar{k}^2 B_{1z}\right) - \nabla^2 V_d\left(\frac{n_{1d}}{n_{0d}}\right) - \nabla^2 \psi_1 = 0} \quad (8)$$

As MHD equation is

$$\frac{\partial B}{\partial t} = \nabla \times (V \times B)$$

Linearizing above equation and using identity

$$\nabla \times (F \times G) = F(\nabla \cdot G) - G(\nabla \cdot F) + (G \cdot \nabla)F - (F \cdot \nabla)G.$$

$\therefore$